



**University
of Victoria**

Graduate Studies

Notice of the Final Oral Examination
for the Degree of Doctor of Philosophy

of

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“Lace Tessellations:
A Mathematical Model for Bobbin Lace
and an Exhaustive Combinatorial Search for Patterns”

Department of Computer Science

Wednesday, August 10, 2016
10:00 A.M.
David Turpin Building
Room A144

Supervisory Committee:

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Dr. Wendy Myrvold, Department of Computer Science, UVic (Member)
Dr. Gary MacGillivray, Department of Mathematics and Statistics, UVic (Outside Member)

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Dr. Jo-Anne Lee, Department of Gender Studies, UVic

Dr. David Capson, Dean, Faculty of Graduate Studies

Abstract

Bobbin lace is a 500-year-old art form in which threads are braided together in an alternating manner to produce a lace fabric. A key component in its construction is a small pattern, called a bobbin lace ground, that can be repeated periodically to fill a region of any size. In this thesis we present a mathematical model for bobbin lace grounds representing the structure as the pair $(\Delta_1(G), \zeta)$ where $\Delta_1(G)$ is a topological embedding of a toroidal 2-regular digraph, G , and ζ is a mapping from the vertices of G to a set of braid words. We explore in depth the properties that $\Delta_1(G)$ must possess in order to produce workable lace patterns. Having developed a solid, logical foundation for bobbin lace grounds, we enumerate and exhaustively generate patterns that conform to that model. We start by specifying an equivalence relation and define what makes a pattern prime so that we can identify unique representatives. We then prove that there are an infinite number of prime workable patterns. One of the key properties identified in the model is that it must be possible to partition $\Delta_1(G)$ into a set of osculating circuits such that each circuit has a wrapping index of $(1; 0)$; that is the circuit wraps once around the meridian of the torus and does not wrap around the longitude. We use this property to exhaustively generate workable patterns for increasing numbers of vertices in G by gluing together lattice paths in an osculating manner. Using a backtracking algorithm to process the lattice paths, we identify over 5 million distinct prime patterns. This is well in excess of the roughly 1,000 found in lace ground catalogues. The lattice paths used in our approach are members of a family of partially directed lattice paths that have not been previously reported. We explore these paths in detail, develop a recurrence relation and generating function for their enumeration and present a bijection between these paths and a subset of Motzkin paths. Finally, to draw out of the extremely large number of patterns some of the more aesthetically interesting cases for lacemakers to work on, we look for examples that have a high degree of symmetry. We demonstrate, by computational generation, that there are lace ground representatives from each of the 17 planar periodic symmetry groups.